Name: **Enrolment No:**	UPES — UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Course: Differential Calculus **Semester: I**
Program: B.Sc. (Hons.) Mathematics by Research **Time: 03 hrs.**
Course Code: MATH1070 **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Find the n^{th} derivative of $\cos^4 x$.	**4**	**CO1**
Q 2	Show that $\lim_{x\to 2} 4x - 5 = 3$ using $\epsilon - \delta$ definition.	**4**	**CO1**
Q 3	Find the length of polar sub-tangent and polar sub-normal for the curve $\frac{2a}{r} = 1 - \cos\theta$.	**4**	**CO2**
Q 4	For the curve $r^2 = b^2 \sec 2\theta$, prove that (i) $\psi = \frac{\pi}{2} - \theta$ (ii) $pr = b^2$ Where ψ be the angle which tangent makes with the x-axis.	**4**	**CO2**
Q 5	Find the equation of the tangent plane and normal line of a surface $f(x,y,z) = x^2 + y^2 + z^2 - 9 = 0$ at point (1,2,4).	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Trace the curve $y^2(a-x) = x^2(a+x)$.	**10**	**CO2**
Q 7	Show that the function $f(x,y) = \begin{cases} x^2 \sin\frac{1}{x} + y^2 \sin\frac{1}{y}, & x \neq 0, y \neq 0 \\ 0, & x = 0, y = 0 \end{cases}$ is differentiable at (0,0).	**10**	**CO3**
Q 8	Find the radius of curvature of the curve $x^2 y = a(x^2 + y^2)$ at point $(-2a, 2a)$.	**10**	**CO2**
Q 9	If $y = \cos(m \sin^{-1} x)$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$ **OR**	**10**	**CO1**

	State Lagrange's Mean Value Theorem and hence prove that $\frac{\cos a\theta - \cos b\theta}{\theta} \leq (b-a)$ if $\theta \neq 0$.		
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	(i) If $x+y+z=u, y+z=uv, z=uvw$ then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$. (ii) Find the maximum value of $u = x^p y^q z^r$ when the variable x, y, z are subject to condition $ax+by+cz=p+q+r$.	**10+10**	**CO4**
Q 11	(i) If $v = (1-2xy+y^2)^{-\frac{1}{2}}$ then find the value of $\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial v}{\partial x}\right] + \frac{\partial}{\partial y}\left(y^2\frac{\partial v}{\partial y}\right)$ (ii) Sketch the level curves of the function $f(x,y) = \sqrt{9-x^2-y^2}$ for the values $k = 0,1,2,3$. **OR** (i) Prove that $f(x,y) = \begin{cases} \frac{x^3y^3}{x^2+y^2}, & x \neq 0, y \neq 0 \\ 0, & x=0, y=0 \end{cases}$ is continuous at origin. (ii) Find the directional derivative of $\phi = xy^2 + yz^2$ at the point $(2,-1,1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.	**10+10**	**CO3**

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