Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Course: Theory of Ordinary Differential Equations **Semester: I**
Program: M.Sc. (Mathematics) **Time: 03 hrs.**
Course Code: MATH7042 **Max. Marks: 100**

Instructions: Attempt all questions from Section A, Section B and Section C. There are internal choices in Questions 9 and 10. Use of a scientific calculator is permitted.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	Show that first-order differential equation $$\left\lvert\frac{dy}{dx}\right\rvert + \lvert y\rvert + 1 = 0$$ has no (real) solutions.	**4**	**CO1**
Q 2	Apply basic existence and uniqueness theorem to show that the following initial value problem has a unique solution on some sufficiently small interval $\lvert x-1\rvert \le h$ about $x_0 = 1$: $$\frac{dy}{dx} = x^2 \sin y; \quad y(1) = -2.$$	**4**	**CO1**
Q 3	Determine whether $x = 0$ is an ordinary point or regular singular point of the differential equation, $$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$	**4**	**CO3**
Q 4	The roots of the auxiliary equation, corresponding to a certain 12^{th}-order homogeneous linear differential equation with constant coefficients are $2, 2, 2, 2, 2, 2, 3+4i,\ 3-4i,\ 3+4i,\ 3-4i,\ 3+4i, 3-4i.$ Write the general solution.	**4**	**CO2**
Q 5	Determine the nature of the critical point $(0,0)$ of the system $$\frac{dx}{dt} = 2x + 4y,$$ $$\frac{dy}{dt} = -2x + 6y.$$	**4**	**CO5**

SECTION B **(4Qx10M= 40 Marks)**			
Q 6	Find the general solution of the Cauchy-Euler differential equation, $$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x, \text{ assume } x > 0.$$	**10**	**CO2**
Q 7	Find the power series solution of the following differential equation about an ordinary point $x = 0$, $$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - xy = 0.$$	**10**	**CO3**
Q 8	Consider the two nonlinear systems, $\frac{dx}{dt} = 8x - y^2,$ $\frac{dy}{dt} = -6y + 6x^2.$ Determine the type and stability of each of the critical points.	**10**	**CO5**
Q 9	Reduce the following differential equation into normal form $$\left(\frac{d^2y}{dx^2} + y\right)\cot x + 2\left(\frac{dy}{dx} + y\tan x\right) = \sec x.$$ Hence obtain the general solution. **OR** Transform the differential equation, $\cos x\frac{d^2y}{dx^2} + \sin x\frac{dy}{dx} - 2y\cos^3 x = 2\cos^5 x,$ into the one having z as independent variable, where $z = \sin x$ and solve it.	**10**	**CO2**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Using Frobenius method, solve the Bessel differential equation, $$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0,$$ taking $2n$ as nonintegral. **OR** Use the method of Frobenius to find the solution near $x = 0$ of the differential equation, $$x(1-x)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0.$$	**20**	**CO3**

Q 11	Find the eigenvalues and the corresponding eigenfunctions of following boundary value problem, $\frac{d^2X}{dx^2} + \lambda X = 0, \quad X(0) = 0$ and $\left[\frac{dX}{dx}\right]_{x=L} = 0.$	**20**	**CO4**