Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Course: **Integral Equations and Calculus of Variations** **Semester: I**
Program: **M. Sc. Mathematics** **Time : 03 hrs.**
Course Code: MATH7043 **Max. Marks: 100**

Instructions: Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (Each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 7 and 10 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Define linear and non-linear integral equations with suitable examples.	**4**	**CO1**
Q 2	Compute iterated kernels (or functions) $K_1(x,t)$ and $K_2(x,t)$ for the integral equation $y(x) = x + \int_0^{1/2} y(t)dt$.	**4**	**CO2**
Q 3	Show that the integral equation $y(x) = \lambda \int_0^1 \sin \pi x \cos \pi t \, y(t)dt$, does not possesses any characteristic number.	**4**	**CO2**
Q 4	Find the extremals of the functional $\int_{x_0}^{x_1} [16y^2 - (y'')^2 + x^2]dx$.	**4**	**CO3**
Q 5	State Hamilton's principle of least action.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Form an integral equation corresponding to the differential equation given by $$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0,$$ with the initial conditions $y(0) = 1, y'(0) = 0$.	**10**	**CO1**
Q 7	Find the Neumann series for the solution of the Volterra integral equation $y(x) = 1 + x + \lambda \int_0^x (x - t) \, y(t)dt$. **OR** With the aid of the resolvent kernel, find the solution of the integral equation $y(x) = \sin x + 2 \int_0^x e^{x-t} \, y(t)dt$.	**10**	**CO2**

Q 8	On what curves can the functional $$V[y(x)] = \int_0^{\pi} [y'^2 - y^2 + 4y\cos x]dx;\ y(0) = 0, y(\pi) = 0$$ be extremized?	**10**	**CO3**
Q 9	Discuss the Jacobi and Legendre conditions for extremum for the functional $$I[y(x)] = \int_0^1 \left[\frac{1}{2}x^2 y'^2 - 2xyy' + y\right] dx;\ u(0) = 0,$$ where $u = \delta y$. Further, derive the extremal satisfying $u(1) = \frac{1}{2}$ and emanating from $(0, 1)$.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Transform the following boundary value problem $$\frac{d^2y}{dx^2} + xy = 1; y(0) = 0, y(1) = 0,$$ into an integral equation. Also, recover the boundary value problem from the integral equation obtained. **OR** Solve the Fredholm integral equation $$y(x) = 1 + \lambda \int_0^1 (x + t)\ y(t)dt,$$ by the method of successive approximations to the third order.	**20**	**CO2**
Q 11	(i) Determine the extremal of the functional $$I[y(x)] = \int_0^{\frac{\pi}{4}} [y''^2 - y^2 + x^2]dx,$$ under the conditions $y(0) = 0, y'(0) = 1, y\left(\frac{\pi}{4}\right) = y'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$. (ii) Prove that the extremal of the isoperimetric problem $$I[y(x)] = \int_1^4 y'^2 dx,$$ with $y(1) = 3, y(4) = 24$ subject to the condition $\int_1^4 y dx = 36$ is a parabola.	**10+10**	**CO4**