Name: **Enrolment No:**	

UPES

End Semester Examination, December 2024

Course: Mathematical Physics **Semester: I**
Program: M.Sc. Physics **Time : 03 hrs.**
Course Code: PHYS7002 **Max. Marks: 100**

Instructions: (a) Answer all questions of Section A.
(b) Answer all questions of Section B. (In one question, internal choice provided).
(c) Answer all questions of Section C. (In one question, internal choice provided).
(d) Non-programmable scientific calculator is allowed.

SECTION A

($5Q \times 4M = 20Marks$)

S. No.		Marks	CO
Q 1	Define Gauss divergence and Stokes' theorems.	**4**	**CO1**
Q 2	Define Hermitian, and skew-Hermitian Matrices.	**4**	**CO1**
Q 3	Obtain p such that the function $f(z)$ expressed in polar coordinates as $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic.	**4**	**CO2**
Q 4	Apply Runge 's Formula (third order) to solve the differential equation $\frac{dy}{dx} = x - y$ subject to $y = 1$ when $x = 1$. (Solve up to $y(0.1), h = 0.1$).	**4**	**CO3**
Q 5	Define Group (G, *) using all the four properties.	**4**	**CO1**

SECTION B

($4Q \times 10M = 40\ Marks$)

Q 6	Show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic function and determine its harmonic conjugate function.	**10**	**CO3**
Q 7	The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men, now 60, at least 7 will live to be 70?	**10**	**CO2**

<table>
<tr><td>Q 8</td><td>Show that $\left\{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}\omega & 0\\0 & \omega^2\end{pmatrix}, \begin{pmatrix}\omega^2 & 0\\0 & \omega\end{pmatrix}, \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & \omega^2\\\omega & 0\end{pmatrix}, \begin{pmatrix}0 & \omega\\\omega^2 & 0\end{pmatrix}\right\}$, where $\omega^3 = 1, \omega \neq 1$ form a group with respect to matrix multiplication.</td><td>10</td><td>CO3</td></tr>
<tr><td>Q 9</td><td>Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule and Simpson's $1/3$ rule.
OR
Apply Lagrange's formula to determine the polynomial $f(x)$ and hence calculate $f(3)$ for
<table><tr><td>x:</td><td>0</td><td>1</td><td>2</td><td>5</td></tr><tr><td>$f(x)$:</td><td>2</td><td>3</td><td>12</td><td>147</td></tr></table></td><td>10</td><td>CO2</td></tr>
<tr><td colspan="4">SECTION-C
$(\boldsymbol{2Q \times 20M = 40\ Marks})$</td></tr>
<tr><td>Q 10</td><td>Analyze and evaluate
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$
which satisfies the conditions: $u(0, y) = u(l, y) = u(x, 0) = 0$,
$and\ u(x, a) = \sin\frac{n\pi x}{l}$.</td><td>20</td><td>CO4</td></tr>
<tr><td>Q 11</td><td>Determine the characteristic equation of the symmetric matrix
$$A = \begin{bmatrix}2 & -1 & 1\\-1 & 2 & -1\\1 & -1 & 2\end{bmatrix}$$
and verify that it is satisfied by A and hence obtain A^{-1}.
Express $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$ in linear polynomial in A.
OR
Determine all the eigen values and eigen vectors of the matrix
$$\mathrm{A} = \begin{bmatrix}-2 & 2 & -3\\2 & 1 & -6\\-1 & -2 & 0\end{bmatrix}$$</td><td>20</td><td>CO2</td></tr>
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