**Name:**

**Enrolment No:**

# UPES

## END Semester Examination, December 2024

**Programme Name : B.Sc.(Mathematics by Research)** **Semester : III**
**Course Name : Real Analysis** **Time : 03 hrs**
**Course Code : MATH2060** **Max. Marks: 100**
**Nos. of page(s) : 02**

**Instructions: All questions are compulsory. There is an internal choice in Q9 and Q11.**

**SECTION A**
**(4 Marks * 5 = 20 Marks)**
**Answer all questions**

| S. No. | | Marks | CO |
|---|---|---|---|
| Q 1 | Consider the series of functions $$\sum_{n=1}^{\infty} \frac{\sin(2nx+5n)}{n(n+1)}, x \in \mathbb{R}$$ Identify the uniform convergence of the series. | **4** | **CO2** |
| Q 2 | Find lower Riemann sum and upper Riemann sum of the function $$f(x) = \begin{cases} -11 & if\ x \in \mathbb{Q} \\ 11 & if\ x \notin \mathbb{Q} \end{cases}.$$ | **4** | **CO1** |
| Q 3 | Give an example of a series which is conditionally convergent but not absolutely. | **4** | **CO3** |
| Q 4 | Show that $f(x) = 1$ is Riemann-integrable in $[0, \frac{\pi}{2}]$ | **4** | **CO1** |
| Q 5 | Let $f_n(x)$ be given by $$f_n(x) = x^n ; n \in \mathbb{N}, x \in [0,1].$$ Find the limit function $f(x)$. Is the convergence uniform? | **4** | **CO2** |

**SECTION B**
**(10 Marks * 4 = 40 Marks)**
**Answer all questions. There is an internal choice in Q9.**

| S. No. | | Marks | CO |
|---|---|---|---|
| Q 6 | Determine the radius of convergence and the exact interval of convergence of the power series $$\sum \frac{(n)(3x+1)^n}{(n+5)(n+9)}.$$ | **10** | **CO3** |

| | | | |
|---|---|---|---|
| Q 7 | For what values of $x$, the series $$\frac{1}{1.2.3}x+\frac{3}{2.3.4}x^2+\frac{5}{3.4.5}x^3+\cdots$$ is convergent? | **10** | **CO3** |
| Q8 | Define $f:[0,1]\to[0,1]$ by $f(x)=\frac{2^k-1}{2^k}$ for $x\in\left[\frac{2^{k-1}-1}{2^{k-1}},\frac{2^k-1}{2^k}\right]$. Find the upper Reimann sum of the given function. | **10** | **CO1** |
| Q 9 | Find the lower Riemann sum of the function $f(x)=sinx\,,x\in\left[0,\frac{\pi}{2}\right]$ <br> **OR** <br> Prove that a monotonic function $f:[a,b]\to\mathbb{R}$ on a compact interval is Riemann integrable. | **10** | **CO1** |
| **SECTION C** <br> **(20 Marks * 2 = 40 Marks)** <br> **Answer all questions. There is an internal choice in Q11.** | | | |
| Q 10 | If the series $$\sum_{n=1}^{\infty}a_n$$ is absolutely convergent. Prove or disprove each of the following. <br> a) $\sum_{m=n}^{\infty}a_m\to 0$ as $n\to\infty$. <br> b) $\sum_{n=1}^{\infty}a_n\sin n$ is convergent. <br> c) $\sum_{n=1}^{\infty}e^{a_n}$ is divergent. <br> d) $\sum_{n=1}^{\infty}a_n^2$ is divergent. | **20** | **CO3** |
| Q 11 | Check the pointwise and uniform convergence of the sequence of functions $$f_n(x)=\frac{nx}{1+n^2x^2}\,;x\in[0,1]$$ **OR** <br> Let $f_n:[2,3]\to[0,1]$ be given by $f_n(x)=(3-x)^n$ for all nonnegative integers $n$. Let $f(x)=\lim_{n\to\infty}f_n(x)$ for $2\le x\le 3$. <br> a) Find the pointwise limit $f(x)$. Is the convergence uniform? <br> b) Check whether the following is true or not $\lim_{n\to\infty}\int_2^3 f'_n(x)\,dx=\int_2^3 f(x)\,dx$. | **20** | **CO2** |