Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

Program: BSc. Hons. Mathematics
Time : 03 hrs.
Course Code: MATH3031 **Max. Marks: 100**

Instructions: All questions are mandatory. There are internal choices in Q 9 of Section B and Q 11 of Section C.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Find all the generators of the group Z_{20}.	**4**	**CO1**
Q 2	Determine all the ring homomorphisms from Z_{12} to Z_{30}.	**4**	**CO2**
Q 3	A finite ring must have a nonzero characteristic. Give reasons to justify this statement.	**4**	**CO2**
Q 4	List all the elements in the quotient ring $Z_5[i]/\langle i+1\rangle$.	**4**	**CO3**
Q 5	In $Z[i]$, show that 3 is irreducible but 2 and 5 are not.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	(a) What is the number of elements of order 15 in Z_{30}. (b) Find $Z(S_3)$ and $C((1\ 3))$ in S_3 where $Z(S_3)$ is the center of S_3 and $C((1\ 3))$ is the centralizer of $(1\ 3)$.	**10**	**CO1**
Q 7	(a) Let F be a finite field with n elements. Prove that $x^{n-1}=1$ for all nonzero x in F. (b) Find the characteristics of the ring $Z_4 \oplus 4Z$.	**10**	**CO2**
Q 8	(a) Show that $R[x]/\langle x^2+1\rangle$ is a field. (b) Show that $\langle x^2+1\rangle$ is not a prime ideal in $Z_2[x]$.	**10**	**CO3**
Q 9	(a) Prove that the set of all polynomials whose coefficients are even is a prime ideal in $Z[x]$. (b) Is the homomorphic image of a principal ideal domain a principal ideal domain? Justify. **OR** (a) Find an ideal I of $Z_8[x]$ such that the factor ring $Z_8[x]/I$ is a field.	**10**	**CO3**

	(b) Determine all the units in $Z[i]$.		
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	(a) Draw the lattice of ideals of Z_{36}. Shows that only maximal ideals of Z_{36} are $< 2 >$ and $< 3 >$. (b) Show that mapping ϕ from $Z[x]$ onto Z given by $\phi(f(x) = f(0)$ is a ring homomorphism with $Ker(\phi) = \langle x \rangle$.	**20**	**CO2**
Q 11	(a) Show that $x^2 + 3x + 2$ has four zeros in Z_{36}. (b) Prove that $Q[x]/\langle x^2 - 1\rangle$ is ring-isomorphic to $Q[\sqrt{2}]$. (c) Show that the polynomial $f(x) = 2x^2 + 4$ is irreducible over Q but reducible Z. **OR** (a) Find all the prime ideals Z_{10}. (b) Construct a field of order 27. (c) Show that any ideal of Z is of the form nZ for some ideal. Classify the maximal ideals of Z.	**20**	**CO3**