Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Program Name: B.Sc. Physics by Research **Semester: VII**

Course Name: CLASSICAL MECHANICS **Time: 03 hrs.**

Course Code: PHYS 4020 **Max. Marks: 100**

Instructions:

1. All questions are compulsory (Q. No. 9 and Q. No. 11 have internal choices).
2. Scientific calculators can be used for calculations.
3. All bold representations are vectors.

SECTION A

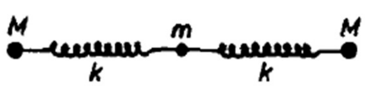
(5Qx4M=20Marks)

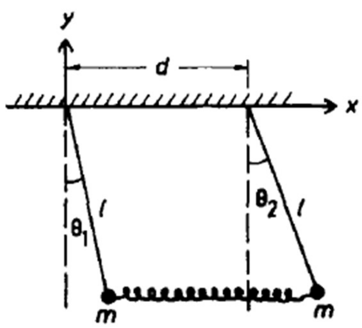
S. No.		Marks	CO
Q 1	State Hamilton's principle of stationary action.	**4**	**CO1**
Q 2	Define cyclic coordinates and their relation to conservation theorems.	**4**	**CO1**
Q 3	Define Poisson brackets for two functions defined on the phase space.	**4**	**CO1**
Q 4	Show that Hermitian matrix has orthogonal eigenvectors for distinct eigenvalues.	**4**	**CO2**
Q 5	A body of rest mass m_0 moving at speed v collides with and sticks to an identical body at rest. What is the mass and momentum of the final clump?	**4**	**CO2**

SECTION B

(4Qx10M= 40 Marks)

Q 6	A particle under the action of gravity slides on the inside of a smooth paraboloid of revolution whose axis is vertical. Using the distance from the axis, r **,** and the azimuthal angle φ **as** generalized coordinates, find (a) The Lagrangian of the system. (b) The generalized momenta and the corresponding Hamiltonian. (c) The equation of motion for the coordinate r as a function of time. (d) If $\frac{d\varphi}{dt} = 0$, show that the particle can execute small oscillations about the lowest point of the paraboloid, and find the frequency of these oscillations.	**10**	**CO3**
Q 7	The transformation equations between two sets of coordinates are $Q = \ln(1 + q^{1/2}\cos p)$ $P = 2(1 + q^{1/2}\cos p)q^{1/2}\sin p$	**10**	**CO3**

	(i) Show directly from these transformation equations that *Q, P* are canonical variables if q, p are. (ii) Show that the function that generates this transformation between the two sets of canonical variables is $$F_3 = -[e^Q - 1]^2 \tan p$$		
Q 8	Consider a particle of rest mass m_0 moving at velocity *v* in the frame *S*. Write down expressions for the components of its energy momentum vector $P = (p_0, p_1)$ in terms of m_0, *v*. Now if you see this particle from a different frame S' moving at the velocity u. What will be its velocity *w* and what will be the components of $P' = (p'_0, p'_1)$ first in terms of *w* and then in terms of *w* written in terms of u and *v*? Show that the primed coordinates are related to the unprimed ones by the same Lorentz Transformation that relates (x_0, x_1) to (x'_0, x'_1).	**10**	**CO2**
Q 9	Discuss the method of calculus of variation and obtain the expression that describes the stationary path. Use it to obtain minimum surface of revolution. **OR** Derive Hamilton's equations of motion starting from a variational principle.	**10**	**CO2**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Consider two particles interacting by way of a central force (potential = *V(*r*)* here **r** is the relative position vector). (i) Obtain the Lagrangian in the center of mass system and show that the energy and angular momentum are conserved. Prove that the motion is in a plane and satisfies Kepler's second law (that **r** sweeps out equal areas in equal times). (ii) Suppose that the potential is V = $kr^2/2$**,** where k is a positive constant, and that the total energy E is known**.** Find expressions for the minimum and maximum values that r will have during the motion.	**15+5 = 20**	**CO2**
Q 11	Consider the longitudinal motion of the system of masses and springs illustrated in the figure below with $M > m$. (i) What are normal mode frequencies of the system? (ii) If the left-hand mass receives an impulse P_0 at t=0, find the motion of the left-hand mass as a function of time. (iii) If, alternatively, the middle mass is driven harmonically at a frequency $\omega_0 = 2\sqrt{\frac{k}{m}}$, will it move in or out of phase with the driving motion? Explain. 	**8+8+4=20**	**CO3**

	OR Two pendulums of equal length l and mass m are coupled by a massless spring of constant k as shown below. The unstretched length of the spring is equal to the distance between the supports. (i)Set up the exact Lagrangian in terms of appropriate generalized coordinates and velocities. (ii) Find the normal coordinates and frequencies of small vibrations about equilibrium. (iii) Suppose that initially the two masses are at rest. An impulsive force gives a horizontal velocity v towards the right to the mass m on the left. What is the motion of the system in terms of the normal coordinates? 	**8+8+4=20**	