Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Course: Commutative Algebra **Semester: VII**
Program: Integrated B.Sc. – M.Sc. in Mathematics **Time: 03 hrs.**
Course Code: MATH3054P **Max. Marks: 100**

Instructions: Attempt all questions from Sections A, B and C. Questions 9 and 11 have an internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	Show that in an Artinian ring, all the prime ideals are maximal.	**4**	**CO3**
Q 2	Prove that a local ring does not contain idempotent elements except 0 and 1.	**4**	**CO1**
Q 3	The integral closure of $K[t]$ is $K[\,t^2, t^3]$, for any field of fractions K – Prove or disprove.	**4**	**CO2**
Q 4	For any primary decomposable ring A, show that radical of A is the smallest prime ideal containing A.	**4**	**CO1**
Q 5	Show that for any set $C = \{\,x \in B \mid x$ is an integral in $B\ over\ A\}$ is a subring of B containing A.	**4**	**CO2**

SECTION B
(4Qx10M= 40 Marks)

S. No.		**Marks**	**CO**
Q 6	Any two bases for a free module M over a commutative ring have the same cardinality.	**10**	**CO2**
Q 7	Let K be the field of fractions and B be a valuation ring of K, then show the following: (i) B is the local ring. (ii) For any ring B' such that $B \subseteq B' \subseteq K$, then show that B' is a valuation ring of K.	**7 + 3**	**CO3**
Q 8	Let A be a ring, A' be an A – algebra. Then A' is finitely generated as an A – algebra and is integral over A iff $A[x_1, x_2, \dots, x_n]$ with all x_i's are integral over A.	**10**	**CO2**
Q 9	A ring R is a Dedekind ring iff R is a Noetherian integrally closed integral domain of dimension 1, so every non-zero prime ideal is maximal. Or State and Prove the Going–Up Theorem.	**10**	**CO1**

SECTION-C (2Qx20M=40 Marks)			
Q 10	Let $A \subseteq B$ be rings. Then establish the following equivalent statements (a) $x \in B$ is integral over A. (b) $A[x]$ is a finitely generated A – module. (c) $\exists$ a finitely generated A – module C containing A and x. (d) $\exists$ a faithful $A[x]$ – module M which is finitely generated.	**5+5+5+5**	**CO2**
Q 11	Let A be a Noetherian ring. Then show that the polynomial ring A[x] is also a Noetherian ring. Or State and Prove the Hilbert Nullstellensatz's theorem.	**20**	**CO3**