Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Course: Functional Analysis **Semester: VII**

Program: Integrated B.Sc.-M.Sc. Mathematics **Time : 03 hrs.**

Course Code: MATH4001 **Max. Marks: 100**

Instructions: There are total eleven questions in two pages. Answer the questions in legible handwriting mentioning solutions to question number properly

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Choose the correct option. i. Dimension of $\mathbb{C}^n$ as a linear space over C is: a) n b) $n+1$ c) n^2 d) $2n$ ii. Consider $f: \mathbb{R} \to \mathbb{R}$. Which of the following is not a linear map? a) $f(x) = x$ b) $f((x) = x^2$ c) $f(x) = 3x + 1$ d) $f(x) = 0$ iii. Which of the following is not a property of norm in general? a) $\|\|x\|\| \geq 0$ b) $\|\|x + y\|\| \leq \|\|x\|\| + \|\|y\|\|$ c) $\|\|kx\|\| = k\ \|\|x\|\|$ d) $\|\|x\|\| = 0$ iff $x = 0$ iv. A complete norm space is known as a: a) Hilbert space b) Compact space c) Banach space d) Euclidean space	**1 x 4**	**CO1**
Q 2	Write the (i) Closed graph theorem, and (ii) Uniform boundedness principle	**4**	**CO2**

Q 3	Let Y be any closed subspace of a Hilbert space H. Then show that $H = Y \oplus Y^{\perp}$.	**4**	**CO3**
Q 4	If X and $Y$are Banach spaces and $T \in B[X, Y]$ is injective and surjective, then $T^{-1} \in B[X, Y]$.	**4**	**CO4**
Q 5	Show that the graph of a linear map P on a Banach space X is closed.	**4**	**CO6**
SECTION B **(4Qx10M= 40 Marks)** **There is an internal choice in Q9**			
Q 6	State and prove the Open mapping theorem.	**10**	**CO2**
Q 7	If Y is a closed subspace of a Hilbert space H, then $Y = Y^{\perp\perp}$.	**10**	**CO4**
Q 8	Let $X = Y \oplus Z$ and P be the projection on Y along Z. Then $T \in BL(X)$ is decomposed by a pair (Y, Z) if and only if $PT = TP$.	**10**	**CO5**
Q 9	For any bounded linear functional f on a normed space X, prove that $\|\|f\|\| = \sup\{\|f(x)\|: \|\|x\|\| \leq 1\} = \sup\left\{\frac{1}{\|\|x\|\|} : f(x) = 1\right\}$. **OR** Prove that a linear functional f on a normed space is bounded functional if and only if f is a continuous functional.	**10**	**CO6**
SECTION-C **(2Qx20M=40 Marks)** **There is an internal choice in Q11**			
Q 10	"Let X be a normed space and let Y be a complete normed space. Then $L(X, Y)$ is a Banach space." Prove it.	**20**	**CO4**
Q 11	State Hahn-Banach theorem. Prove that a linear functional on a normed space is a bounded functional if and only if it is continuous functional. **OR** Show that for any x, y in an inner product space X, $< x, y > = \frac{1}{4}\left(\|\|x+y\|\|^2 - \|\|x-y\|\|^2 + \|\|x+iy\|\|^2 - \|\|x-iy\|\|^2\right)$.	**20**	**CO5**