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| **Name:**<br>**Enrolment No:** | |

**UPES**

**End Semester Examination, Dec 2024**

**Course: Operations Research** **Semester: VII**

**Program: Int. B.Sc.-M.Sc. (Mathematics)** **Time : 03 hrs.**

**Course Code: MATH4002** **Max. Marks: 100**

**Instructions: Read the following instructions carefully:**
**Mention Name and Roll No. at the top of the question paper.**
**Attempt all questions.**
**Q. No. 9 & 11 has an internal choice.**
**Use of scientific calculator is allowed.**

**SECTION A**
**(5Qx4M=20Marks)**

| S. No. | | Marks | CO |
|---|---|---|---|
| Q 1 | Show that the set $A = \{(x, y): 0 \leq y \leq 5$ when $0 \leq x \leq 2$ and $3 \leq y \leq 5$ when $2 \leq x \leq 7\}$ is not a convex set. | **4** | **CO1** |
| Q2 | A tape recorder company manufactures models A, B and C, which have profit contributions per unit of Rs 15, Rs 40, and Rs 60, respectively. The weekly minimum production requirements are 25 units for model A, 130 units for model B and 55 units for model C. Each type of recorder requires a certain amount of time for the manufacturing of the component parts for assembling and for packaging. Specifically, a dozen units of model A require 4 hours for manufacturing, 3 hours for assembling and 1 hour for packaging. The corresponding figures for a dozen units of model B are 2.5, 4 and 2 and for a dozen units of model C are 6, 9 and 4. During the forthcoming week, the company has available 130 hours of manufacturing, 170 hours of assembling and 52 hours of packaging time. Formulate the problem as an LP model so as to maximize the total profit of the company. | **4** | **CO1** |
| Q3 | 800 units of certain item of stock are needed over each year period. If the unit cost is Rs. 400 and the cost of each order is Rs. 150, and the carrying cost is 1.5% of the cost of each unit. Determine the Economic Order Quantity (EOQ) for this item and the number of orders per year. | **4** | **CO3** |
| Q4 | Explain the components of Kendall's notation *A/B/C: D/E* for representing a queuing model. Also, present one supporting example. | **4** | **CO3** |

<table>
<tr><td>Q5</td><td>For the following cost (costs in Rs. per unit) matrix:<br><br><table><tr><td></td><td>$D_1$</td><td>$D_2$</td><td>$D_3$</td><td>$D_4$</td><td>$D_5$</td><td>Supply</td></tr><tr><td>$S_1$</td><td>5</td><td>8</td><td>6</td><td>6</td><td>3</td><td>8</td></tr><tr><td>$S_2$</td><td>4</td><td>7</td><td>7</td><td>6</td><td>5</td><td>5</td></tr><tr><td>$S_3$</td><td>8</td><td>4</td><td>6</td><td>6</td><td>4</td><td>9</td></tr><tr><td>Demand</td><td>4</td><td>4</td><td>5</td><td>4</td><td>8</td><td></td></tr></table>Determine an initial basic feasible solution for the given TP using North-West Corner Method (NWCM).</td><td>4</td><td>CO2</td></tr>
<tr><td colspan="4">SECTION B<br>(4Qx10M= 40 Marks)</td></tr>
<tr><td>Q6</td><td>For the following LPP:<br>$$\text{Max } Z = 4x_1 + 6x_2 + 2x_3$$<br>$$\text{s.t. } x_1 + x_2 + x_3 \leq 3,$$<br>$$x_1 + 4x_2 + 7x_3 \leq 9,$$<br>$$x_1, x_2, x_3 \geq 0.$$<br>The optimal table is given below:<table><tr><td>$X_B$</td><td>b</td><td>$x_1$</td><td>$x_2$</td><td>$x_3$</td><td>$s_1$</td><td>$s_2$</td></tr><tr><td>$x_1$</td><td>1</td><td>1</td><td>0</td><td>-1</td><td>4/3</td><td>-1/3</td></tr><tr><td>$x_2$</td><td>2</td><td>0</td><td>1</td><td>2</td><td>-1/3</td><td>1/3</td></tr><tr><td colspan="2">$Z_j - C_j$</td><td>0</td><td>0</td><td>6</td><td>10/3</td><td>2/3</td></tr></table>Determine the range of coefficient $c_1$ of $x_1$ in the objective function such that the current optimal solution remains unchanged.</td><td>10</td><td>CO1</td></tr>
<tr><td>Q7</td><td>A company has 4 machines to do 3 jobs. Each job can be assigned to one and only machine. The cost of each job on each machine is given in the following table:<table><tr><td rowspan="4">Jobs</td><td colspan="5">Machines</td></tr><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>A</td><td>18</td><td>24</td><td>28</td><td>32</td></tr><tr><td>B</td><td>8</td><td>13</td><td>17</td><td>19</td></tr><tr><td>C</td><td>10</td><td>15</td><td>19</td><td>22</td></tr></table>What are job assignments which will minimize the cost?</td><td>10</td><td>CO2</td></tr>
<tr><td>Q8</td><td>A salesman needs to visit 4 cities: A, B, C, and D. The distance (in kilometers) between each pair of cities is given in the following table:<br><br><table><tr><td>City</td><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td>A</td><td>0</td><td>10</td><td>15</td><td>20</td></tr><tr><td>B</td><td>10</td><td>0</td><td>35</td><td>25</td></tr><tr><td>C</td><td>15</td><td>35</td><td>0</td><td>30</td></tr><tr><td>D</td><td>20</td><td>25</td><td>30</td><td>0</td></tr></table>Find the shortest possible route that the salesman can take to visit all cities once and return to the starting city.</td><td>10</td><td>CO2</td></tr>
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|---|---|---|---|
| Q9 | Obtain the optimal solution for the following LPP:<br>Max $Z = 3x_1 + 2x_2 + x_3$<br>s. t. $2x_1 + 5x_2 + x_3 = 12$,<br>$3x_1 + 4x_2 = 11$,<br>$x_2, x_3 \geq 0$, and $x_1$ unrestricted.<br>**OR**<br>Solve the following LPP using simplex method:<br>Max $Z = 3x_1 + 9x_2$<br>s. t. $x_1 + 4x_2 \leq 8$,<br>$x_1 + 2x_2 \leq 4$,<br>$x_1, x_2 \geq 0$. | **10** | **CO4** |
| **SECTION-C**<br>**(2Qx20M=40 Marks)** | | | |
| Q10 | a) Derive an expression for EOQ for deterministic inventory model with allowable shortages.<br>b) What are the main costs associated with inventory? Explain briefly the difference between ordering cost and holding cost. | **15+5** | **CO3** |
| Q11 | A self-service canteen employs one cashier at its canteen counter. The customers are to get their tokens for the dishes earlier. 10 customers arrive on an average every 7 minutes, while the cashier can serve 12 customers in 7 minutes. Assuming Poisson distribution for arrival rate and Exponential distribution for service, find<br>a) Average number of customers in the system.<br>b) Average number of customers in waiting line or average length of the waiting line.<br>c) Average time a customer spends in the system.<br>d) Average time a customer waits before being served.<br>**OR**<br>A single-server ticket counter has an arrival rate of 20 customers per hour and the server can serve 30 customers per hour. Assume the system follows the M/M/1 queuing model. Calculate the following:<br>a) The probability that the server is free (i.e., there are no customers in the system).<br>b) The probability that there are no customers in the queue.<br>c) The probability that there are exactly 5 customers in the system.<br>d) The probability that there are at least 2 customers in the system. | **5+5<br>+<br>5+5** | **CO3** |